LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - STATISTICS

SECOND SEMESTER – APRIL 2013

ST 2962 - MODERN PROBABILITY THEORY

Date : 07/05/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART A

Answer all questions. **(10 x 2 = 20 marks)**

1. Define measurable sets.
2. Show that for any distribution $\int_{-\infty}^{\infty}\left(1-\frac{x^2}{t^2}\right) dF(x) \leq \int_{-t}^{t} dF(x)$
3. Write the properties of a distribution function.
4. Define WLLN and SLLN.
5. Verify that existence of r^{th} absolute moment ⇒ the existence of absolute moment of all lower orders.
6. Prove that the Lebesgue outer measure of any countable set is zero.
7. In how many ways can 10 adults and 5 children stand in a circle so that no 2 children are next to each other.
8. State C_r – inequality.
9. Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then show that $X_n + Y_n \xrightarrow{P} X+Y$.
10. State Liapounous Central Limit Theorem.

PART B

Answer any five questions. **(5 X 8 = 40 marks)**

11. If E_1 and E_2 are measurable so is $E_1 \cup E_2$?
12. If $\{E_n\}$ is a sequence of measurable sets with $E_{n+1} \subset E_n$ for each n and let μ(E_1) is finite then μ (lim E_n) = lim μ(E_n) .
13. Let X ≥ 0 and let F be its distribution function , then show that

    E(X) <∞ ⇔ $\int_o^{\infty}[1-F(x)] < \infty$
14. Obtain the characteristic function of F where

$$F = \begin{cases} 0 & ; \quad x < 0 \\ \frac{x^2}{2} & ; \quad 0 \leq x < 1 \\ 1 - x + \frac{x^2}{2} & ; \quad 1 \leq x < 2 \\ 1 & ; \quad x \geq 2 \end{cases}$$

Hence obtain the r^{th} moment of the random variable about the origin.

15. Prove that $X_m - X_n \xrightarrow{P} 0 \Leftrightarrow X_n \xrightarrow{P} X$ where X is some random variable.

16. Prove that $X_n \xrightarrow{r} X \Rightarrow E|X_n|^r \to E|X|^r$

17. Determine whether WLLN holds for the following sequence of independent random variable :

a) $P[X_n = n] = \frac{1}{2}n^{-\lambda} = P[X_n = -n]$, $P[X_n = 0] = 1 - n^{-\lambda}$

b) $P\left[X_n = {n}/{\log n}\right] = {\log n}/{2n} = P\left[X_n = -{n}/{\log n}\right]$

18. Show that if $X_n \xrightarrow{P} C \Rightarrow F_n(x) \to 0$ for X < c and $F_n(x) \to 1$ for X> c and conversely.

PART C

Answer two questions. **(2 X 20 = 40 marks)**

19. a) Define convergence in probability. State and prove the necessary and sufficient condition for convergence in probability. (10 marks)

b) Prove that $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{P} X$ (5 marks)

c) If $X_n's$ are independent and $X_n \xrightarrow{a.s.} 0$ then $\sum P[|X_n| \geq c] < \infty$ whatever be c > 0 , finite . (5 marks)

OR

d) State and Prove Lindberg – Levy central limit theorem. (10 marks)

e) Let $\{X_n\}$ be any sequence of variates then $S_n = X_1 + X_2 + \cdots + X_n$ and $Y_n = \frac{S_n - E(S_n)}{n}$. Derive the necessary and sufficient condition for the sequence $\{X_n\}$ to satisfy the WLLN is that $E\left[\frac{Y_n^2}{1+Y_n^2}\right] \to 0$ as $n \to \infty$. (10 marks)

20. a) State and Prove the properties of a distribution function . (10 marks)

b) For the distribution function F given below

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ x & ; \quad 0 \le x < \frac{1}{2} \\ x^2 + \frac{3}{8} & ; \quad \frac{1}{2} \le x < \sqrt{\frac{5}{8}} \\ 1 & ; \quad x \ge \sqrt{\frac{5}{8}} \end{cases}$$

Show that this is a distribution function . Also obtain the discontinuity points and decomposed into two distribution functions . (10 marks)

OR

c) State and Prove that mean deviation is least when taken about the median. (10 marks)

d) State and Prove Holder's inequality and hence Schwarz inequality. (10 marks)